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HYDROELASTIC EFFECTS IN THE AORTA BIFURCATION ZONE

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Translation of "Gidrouprugiyе yavleniya v zone bifurkatsii aorty,"  
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16. Abstract Mathematical analysis is made of the mechanical behavior of the vessels and blood at the point of aortic bifurcation, using a homogeneous single-layer channel as a model of the aorta and allowance for the fact that the aortic intima is considerably less rigid than the other coats. For analysis of blood flow in the major arteries, the blood is treated as a viscous Newtonian fluid whose movements are described by Navier-Stokes equations and a continuity equation. Blood flow dynamics at the aortic bifurcation are discussed on the basis of the results.			
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## HYDROELASTIC EFFECTS IN THE AORTA BIFURCATION ZONE

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According to data from autopsies and cardiovascular surgery, /164  
the process of sclerotic plaque formation and embrittlement in persons of all ages, but chiefly in the elderly, frequently occurs where the major vessels ramify, especially at the aortic bifurcation. Serious diseases resulting in inadequate blood supply to the legs are associated with these processes.

Analysis of the hydroelastic aspects of such phenomena would lead to a better understanding of their origins. One obvious cause of atherosclerotic plaques, most often encountered at the edges of the bifurcation, is the formation of closed eddies where other major vessels branch off from the aorta. Combined solution of hydrodynamic and elasticity equations is required for analysis of the mechanical behavior of blood and vessels at the bifurcation. Our work [1] has shown that, in general, a blood vessel should be treated as a multi-layered envelope, one layer of which -- the muscular -- through active contraction exerts a substantial influence upon the initial (for each given hemodynamic problem) condition of the vessel-envelope.

Structurally, the aorta is an elastic-type vessel. The inner coat of the aorta (the intima) is distinguished by its complex structure and, in comparison with the intima of smaller-caliber vessels, its relative thickness. The middle coat of the aorta is made up of dozens of alternating layers of elastic and smooth muscle tissue, and its external coat is comparatively poorly developed. Bearing in mind that the intima is considerably less rigid than the other coats, while the middle coat, because of its large number of layers, is essentially homogeneous, we can use a single-layer envelope as a model of the aorta. Similarly [2], we shall write the equation

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\*Numbers in the margin indicate pagination in the foreign text.

of motion for it as:

$$\frac{\partial N_x}{\partial x} + \frac{\partial T}{\partial y} - k_x \left( \frac{\partial M_x}{\partial x} + \frac{\partial H}{\partial y} \right) + p_x - \rho h \frac{\partial^2 u}{\partial t^2} = 0; \quad (1)$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial T}{\partial x} - k_y \left( \frac{\partial M_y}{\partial y} + \frac{\partial H}{\partial x} \right) - \rho h \frac{\partial^2 v}{\partial t^2} = 0; \quad (2)$$

$$\begin{aligned} \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + k_x N_x + k_y N_y + \frac{\partial}{\partial x} \left( \frac{\partial \omega}{\partial x} N_x + \frac{\partial \omega}{\partial y} T \right) + \\ + \frac{\partial}{\partial y} \left( \frac{\partial \omega}{\partial y} N_y + \frac{\partial \omega}{\partial x} T \right) + p_z - \rho h \frac{\partial^2 \omega}{\partial t^2} = 0. \end{aligned} \quad (3)$$

where  $x$ ,  $y$ , and  $z$  are the coordinates for the median surface of the /165 envelope. Displacements in the  $x$ ,  $y$ , and  $z$  directions are correspondingly designated as  $u$ ,  $v$ , and  $w$ , median surface stress as  $N_x$ ,  $N_y$ , and  $T$ , and the bending and torque moments as  $M_x$ ,  $M_y$ , and  $H$ . The density and thickness of the envelope material are  $\rho$  and  $h$ ;  $k_x$  and  $k_y$  are the median surface curvatures.

Now let's deal with the blood flow equations. A number of models are known to describe blood rheology in the cardiovascular system [3]. When analyzing blood flow in the major vessels, we may consider the blood a viscous Newtonian fluid whose movement is described by the Navier-Stokes equations

$$\begin{aligned} \frac{\partial v_x}{\partial t} + v_r \frac{\partial v_x}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_x}{\partial \theta} + v_x \frac{\partial v_x}{\partial x} = - \frac{1}{\rho_f} \frac{\partial p}{\partial x} + \frac{v}{3} \frac{\text{div } \vec{v}}{\partial x} + \\ + v \left( \frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v_x}{\partial \theta^2} + \frac{\partial^2 v_x}{\partial x^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right) \end{aligned} \quad (4)$$

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and the continuity equation

$$\frac{\partial \rho_f}{\partial t} + \frac{\partial(\rho_f v_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho_f v_\theta)}{\partial \theta} + \frac{\partial(\rho_f v_x)}{\partial x} + \frac{\partial(\rho_f v_r)}{\partial r} = 0. \quad (5)$$

where  $x$ ,  $r$ , and  $\theta$  are fluid particle coordinates;  $v_x$ ,  $v_r$ , and  $v_\theta$  are its velocity projections;  $\nu = \mu/\rho_f$  is the kinematic-viscosity coefficient;  $\rho_f$  is the fluid density. The equation of state  $p = p(\rho_f)$  should be added to these equations.

Boundary conditions for the fluid at the deformed walls of the vessel are:

$$v_r = \frac{\partial w}{\partial t}; \quad v_x = \frac{\partial u}{\partial t}; \quad v_\theta = \frac{\partial v}{\partial t}. \quad (6)$$

The forces exerted by the flowing blood on the walls of the aorta are determined using the relationships

$$p_x = \nu \rho_f \left( \frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x} \right); \quad p_z = p - 2\nu \rho_f \frac{\partial v_r}{\partial r}. \quad (7)$$

As can be seen in equations (1) - (3), we shall disregard the force  $p_y$ .

The stationary blood flow through the bifurcation during diastole may be taken as the original state of the system. Combined integration of the system of equations for the envelope (1) - (3) and the hydrodynamic equations (4) and (5) under the corresponding envelope and fluid resistance conditions (6) and (7) should be performed, giving the changes in pressure at the entrance to the area of the aorta under consideration at the bifurcation and the blood flow during systole. These "entry" functions may be provided as based upon measurement data which are available in the literature.

The difficulties associated with integrating these equations for a non-symmetrical area and in the event of a non-stationary process are obvious. They can only be surmounted by using numerical methods /166 and a large digital computer.

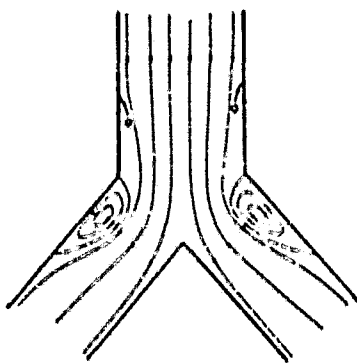


Figure 1

As a first approximation, we can consider the problem of fluid flow in a similarly shaped smooth channel, which will provide a tentative description of blood flow through the bifurcation. This problem is considered [5] with the assumption that the channel walls are rigid, Navier-Stokes equations for an incompressible fluid are transformed into the eddy transfer equation [6]:

$$\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \nabla \psi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \nabla \psi}{\partial y} = \nu \nabla^2 \psi, \text{ etc}$$

where  $\psi$  is the flow function associated with the fluid velocity ratios

$$v_x = \frac{\partial \psi}{\partial y}; \quad v_y = -\frac{\partial \psi}{\partial x}.$$

When velocity changes are uneven, a closed eddy gradually forms at the external angle of the bifurcation near the entrance to the large channel, but it substantially affects the primary flow zone as well (fig. 1). The most intense fluid particle rotation is found at the outside of the streamline, going away from the separation point.

As the current develops in time, the streamline nearest the wall gradually diverges from it, while the separation point (designated in fig. 1 by O) shifts upward somewhat along the flow. A low pressure area forms at the center of the closed eddy. Of extreme importance in this case are the high shift stresses in the area where the closed eddy forms.

The sclerotic phenomena mentioned above are obviously associated with this condition.

We should note that similar processes are observed in the ducts of large hydraulic systems, and they consist of salt deposition on the walls where the duct systems branch.

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